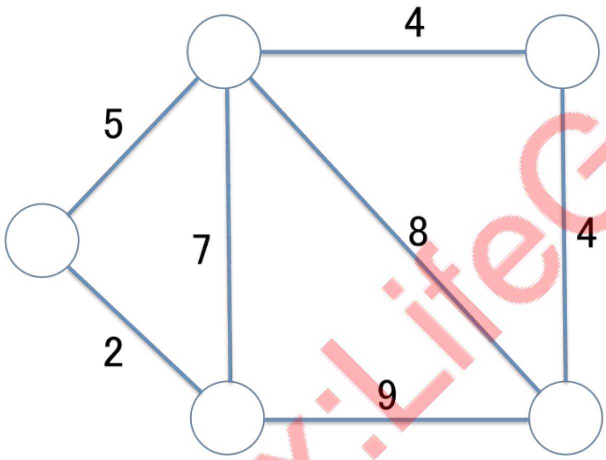
Question 10

Answer the following questions regarding graphs.

(1) Prove that the sum of the vertex degrees of an undirected graph is equal to the number of edges times two.

(2) Prove the following proposition: If a graph $G(V, E)$ with vertex set V and edge set E is a tree, $|E| = |V| - 1$.

(3) Given an undirected graph $G(V, E)$ and a set of edges $T \subseteq E$, the graph $S(V, T)$ is called a spanning tree, if S is a tree. Among all spanning trees of a weighted graph, those with the minimum sum of weights are called minimum spanning trees. Show a minimum spanning tree of the following graph.



(4) Assume that $S(V, T)$ and $S'(V, T')$ are different spanning trees of graph G. Prove the following proposition: For any edge $e' \in T' - T$, there is an edge $e \in T - T'$ such that $(T - \{e\}) \cup \{e'\}$ forms a spanning tree.

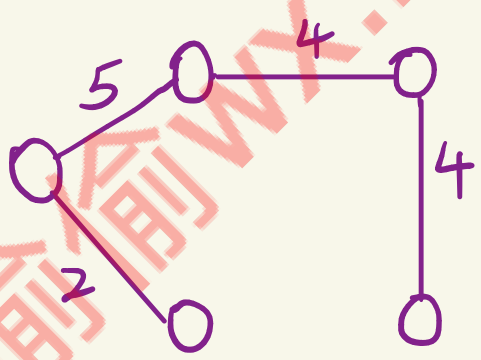
(1) given an undirected graph $G(V,E)$, each edge connects 2 vertices.

$\Rightarrow$ sum of indegree = sum of outdegree $= |E|$

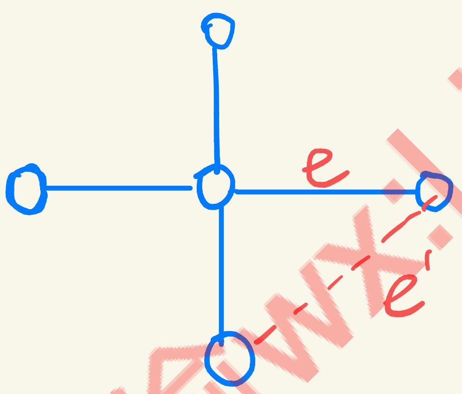
$\Rightarrow$ sum of vertex degrees $= 2|E|$

(2) given a tree graph $G(V,E)$, there is always one edge connecting each node except the tree node. Thus $|E| = |V| - 1$

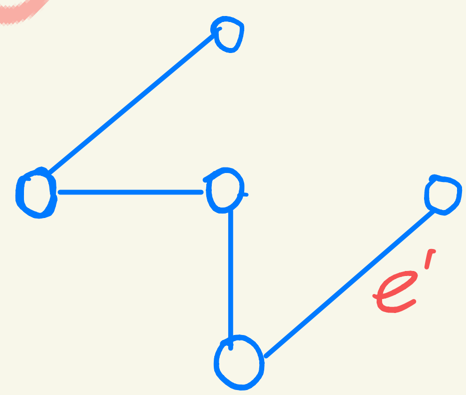
(3)



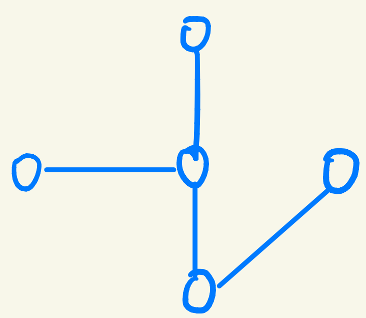
(4) given 2 different spanning trees $S(V,T)$ and $S'(V,T')$, for any edge $e' \in T'-T$, add e' to $S(V,T)$ and $S(V,T)$ forms a circle. The nodes in $S(V,T)$ which e' connects to has 2 degrees, then delete the node's former edge e. As each edge connects different pairs of nodes, it can be easily drawn that $e \in T-T'$, and $(T-\{e\}) \cup \{e'\}$ forms a spanning tree.



$S(V,T)$



$S(V,T')$



$(T-\{e\}) \cup \{e'\}$