Question is translated in English in the section below; this translation is given for reference only.

Answer the following questions about languages $L_1 = \{a(a|b)^n \mid n \geq 0\}$ and $L_2 = \{(a|b)^n b \mid n \geq 0\}$ on an alphabet $\Sigma = \{a, b\}$, where $(a|b)^n$ denotes n repetitions of a or b.

Q.1 Write the production rule set P_1 of the grammar $G_1 = (\{S\}, \Sigma, P_1, S)$ generating L_1, where S is the start symbol.

NFA

Q.2 Write the transition table of a nondeterministic finite automaton M_3 accepting $L_3 = L_1 \cap L_2$ and consisting of three states including the start state q_1 and a final state q_3.

Q.3 Write the transition table of a deterministic finite automaton with the smallest number of states equivalent to the automaton M_3. Here the transition table of a deterministic finite automaton must explicitly specify all the transitions.

Q.4 A subset of a regular language is not always a regular language. Demonstrate this by taking L_3 and $L_4 = \{a^m b^m \mid m \geq 1\}$ as examples.

Q.5 Answer whether $L_5 = \{a^k b^k c^k \mid k \geq 1\}$ is a context-free language or not. Then prove it using the pumping lemma ($\boldsymbol{uvwxy}$ theorem).

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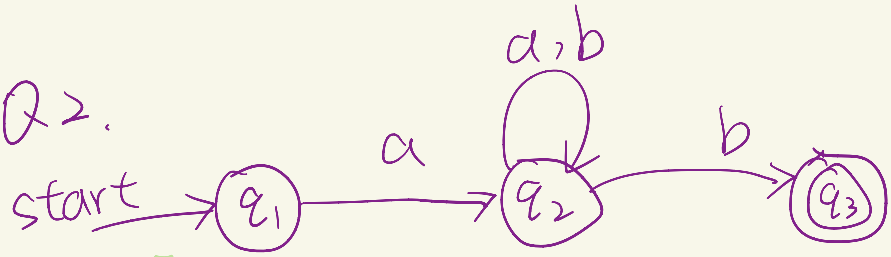
Q1. $S \to Sa \mid Sb \mid a$

Q2.



	a	b
$\to q_1$	$\{q_2\}$	ϕ
q_2	$\{q_2\}$	$\{q_2, q_3\}$
$* q_3$	ϕ	ϕ

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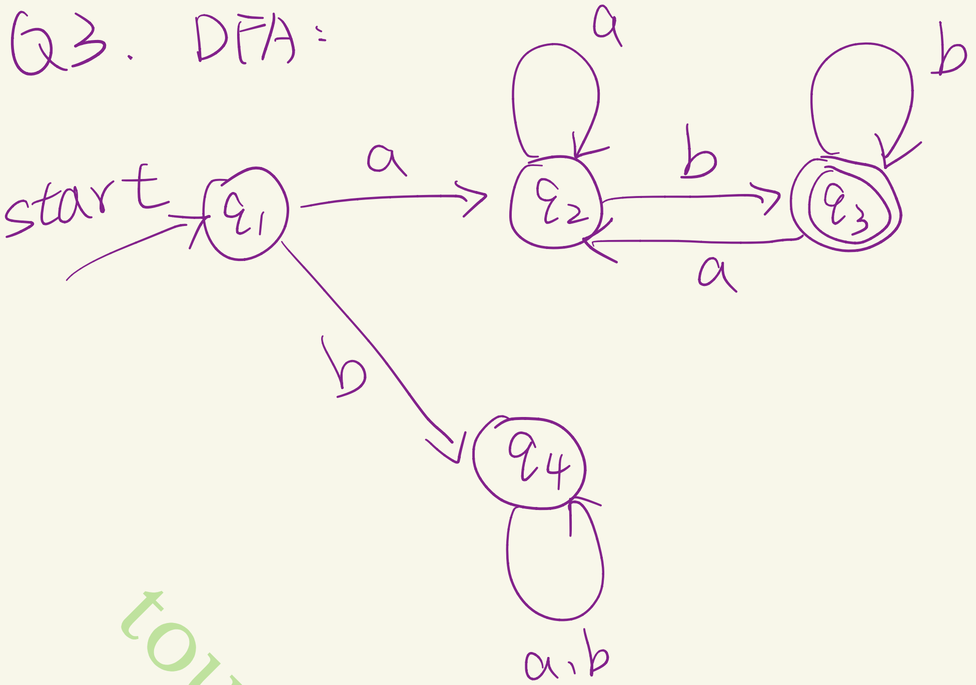
Q3. DFA:



	a	b
$\rightarrow q_1$	q_2	q_4
q_2	q_2	q_3
$*q_3$	q_2	q_3
q_4	q_4	q_4

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Q4. $L_3 = \{ a(a|b)^n b \mid n \geq 0 \}$. thus L_3 is regular language. $L_4 = \{ a^m b^m \mid m \geq 1 \}$ is a subset of L_3.

suppose L_4 is regular language, then there will be $w = a^N b^N = xyz$, $\begin{cases} xy^i z \in L_4, \forall i \in [0,1,2,\cdots] \\ |xy| \leq N \\ |y| > 0 \end{cases}$

let $y = a^k$ ($0 < k \leq n$)

$\Rightarrow$ $xy^0 z = a^{N-k} b^N \notin L_4$ thus L_4 is not regular language, though $L_4 \subseteq L_3$.

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Q5: suppose L_5 is context-free language that there will be $w = a^N b^N c^N = uvwxyz$

$$\begin{cases} |vwx| \le N \\ |vx| > 0 \\ uv^i wx^i y \in L_5, \ \forall i \ge 0 \end{cases}$$

$$\Rightarrow \quad vwx = \begin{cases} a^k \\ b^k \\ c^k \\ a^t b^{k-t} \\ b^t c^{k-t} \end{cases} \quad (0 < k \le N)$$

it can be easily drawn from 5 conditions above that $uv^0 wx^0 z \notin L_5$, thus L_5 is not context-free language.

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