We consider a grammar $G = (\Sigma, N, P, S)$, where Σ, N, P, and S are a finite set of terminal symbols, a finite set of nonterminal symbols, a finite set of production rules, and the start symbol, respectively.

Q.1 The following is a description on context-free grammar. Fill the blanks (1), (2), and (3).

> A grammar G is a context-free grammar when, for each production rule $\alpha \to \beta$ in P, $\alpha \in$ $\boxed{\quad(1)\quad}$ and $\beta \in$ $\boxed{\quad(2)\quad}$. A language L on $\boxed{\quad(3)\quad}$ is a context-free language when it is generated by a context-free grammar G.

Q.2 Prove that $L_{ab} = \{a^m b^m c^n \mid m, n > 0\}$ and $L_{bc} = \{a^m b^n c^n \mid m, n > 0\}$ are context-free languages.

Q.3 Prove that the class of context-free languages is closed under concatenation $L_1 \cdot L_2$ by constructing $G_3 = (\Sigma, N_3, P_3, S_3)$ from the context-free grammars $G_1 = (\Sigma, N_1, P_1, S_1)$ and $G_2 = (\Sigma, N_2, P_2, S_2)$ generating L_1 and L_2, respectively.

Q.4 Prove that the class of context-free languages is closed under union $L_1 \cup L_2$ by constructing $G_3 = (\Sigma, N_3, P_3, S_3)$ from the context-free grammars $G_1 = (\Sigma, N_1, P_1, S_1)$ and $G_2 = (\Sigma, N_2, P_2, S_2)$.

Q.5 $L_{abc} = \{a^k b^k c^k \mid k > 0\}$ is not a context-free language. By using this fact prove that the class of context-free languages is not closed under complement $\overline{L}$.

Q.6 Prove that the class of context-free languages is not closed under difference $L_1 - L_2$.

Q1 (1) N

(2) $(N+\Sigma)^*$

(3) $\{ w \in \Sigma^* \mid S \overset{*}{\underset{G}{\Rightarrow}} w \}$

Q.2 : the production rule of L_{ab}:

$$S \rightarrow AB$$
$$A \rightarrow aAb \mid ab$$
$$B \rightarrow cB \mid c$$

the production rule of L_{bc}:

$$S \rightarrow AB$$
$$A \rightarrow aA \mid a$$
$$B \rightarrow bBc \mid bc$$

Because there exists CFG that generates L_{ab} & L_{bc}, L_{ab} & L_{bc} are context free languages.

Q3. we can construct a CFG G_3 that generates $L_1 \circ L_2$:

$$G_3 = (\Sigma,\ N_1 + N_2,\ P_1 + P_2 + \{S_3 \to S_1 S_2\},\ S_3)$$

Q4. we can construct a CFG G_3 that generates $L_1 \cup L_2$:

$$G_3 = (\Sigma,\ N_1 + N_2,\ P_1 + P_2 + \{S_3 \to S_1 | S_2\},\ S_3)$$

Q5: $L_{abc} = L_{ab} \cap L_{bc}$, L_{abc} is not a context-free language, L_{ab} & L_{bc} is context-free language

$\Rightarrow$ context-free language is not closed under $\cap$.

In Q4 we know context-free language is closed under $\cup$.

Because $L_{abc} = \overline{\overline{L_{ab} \cap L_{bc}}} = \overline{\overline{L_{ab}} \cup \overline{L_{bc}}}$ is not closed.

$\Rightarrow$ context-free language is not closed under $\overline{\ \ }$.

Q6. Suppose 2 context free language L_1 & L_2.

Let $L_3 = L_1 - L_2 = L_1 \cap \overline{L_2}$

in Q5 we know context-free language is not closed under $\overline{L}$ and $\cap$, thus L_3 is not context-free language.

$\Rightarrow$ context-free language is not closed under $-$.