Japanese version shall be the authorized version; the English translation for reference only.

専門基礎 A

A-1, A-2, A-3, A-4, A-5, A-6, A-7, A-8, A-9 の9問から**4問**を選択して解答せよ。

Problem Set A

Choose and answer **4 questions** out of A-1, A-2, A-3, A-4, A-5, A-6, A-7, A-8, and A-9.

A - 1

下記のすべての問に答えよ.
Answer all the following questions.

(1) 次の関数 $f(x)$ について以下の問に答えよ.
Answer the following questions with respect to function $f(x)$.

$$f(x) = \sqrt{1-x^2} + \arcsin x \quad (-1 \leq x \leq 1)$$

(a) 導関数 $f'(x)$ を求めよ.
Derive the derivative $f'(x)$.

(b) $f'(1)$ の値を求めよ.
Find the value $f'(1)$.

(c) $f(x)$ の概形を図示せよ.
Sketch $f(x)$.

(2) $y = x$ と $y = x^2$ の 2 つの曲線で囲まれる領域を D として，次の積分 I を求めよ.
Let D be the region that is enclosed by the two curves $y = x$ and $y = x^2$.
Evaluate the following integral I.

$$I = \iint_D x^2 + y^2 \, dxdy$$

(3) 次の行列 A の固有値および固有ベクトルを求めよ．ただし x は実数とする.
Find the eigenvalues and eigenvectors of the following matrix A, where x is a real number.

$$A = \begin{pmatrix} e^x & e^{-x} \\ e^{-x} & e^x \end{pmatrix}$$

(1) (a) Let $\arcsin x = y \Rightarrow x = \sin y$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

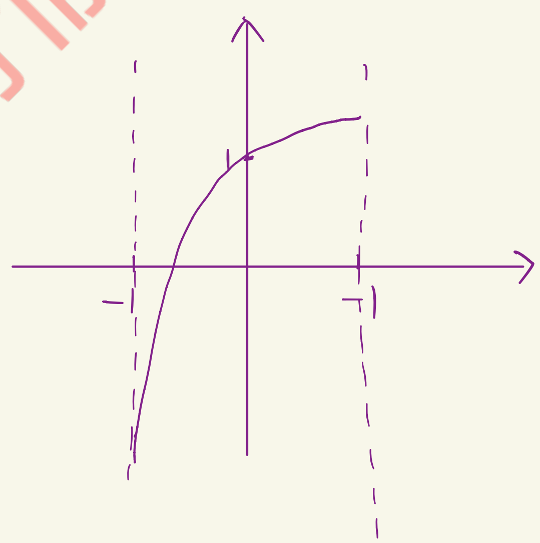
$$f'(x) = \frac{-x}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} = \sqrt{\frac{1-x}{1+x}}, \quad -1<x<1$$

$$f'(x) = \begin{cases} \sqrt{\frac{1+x}{1-x}}, & -1<x<1 \\ \text{No existence}, & x=\pm 1 \end{cases}$$

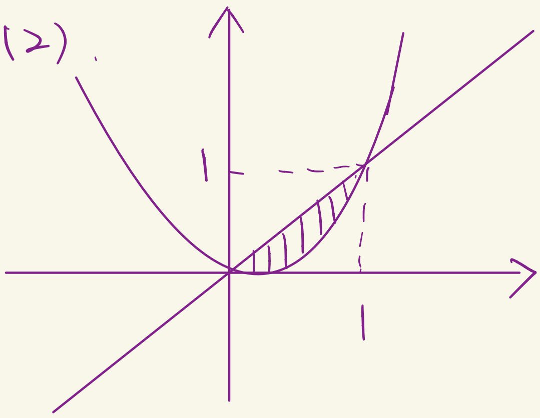
(b)
$$f'(1) = \lim_{\varepsilon\to 0^+} \frac{f(1)-f(1-\varepsilon)}{\varepsilon} = \lim_{\varepsilon\to 0^+} \frac{\frac{\pi}{2} - \sqrt{2\varepsilon-\varepsilon^2} - \arcsin(1-\varepsilon)}{\varepsilon}$$

$$= \lim_{\varepsilon\to 0^+} \frac{\frac{\varepsilon-1}{\sqrt{2\varepsilon-\varepsilon^2}} + \frac{1}{\sqrt{2\varepsilon-\varepsilon^2}}}{1} = \lim_{\varepsilon\to 0^+} \frac{\varepsilon}{\sqrt{2\varepsilon-\varepsilon^2}} = 0$$

(c)



(2)



$$I = \iint_D x^2+y^2 \, dx\,dy = \int_0^1 \int_{x^2}^{x} x^2+y^2 \, dy\,dx$$

$$= \int_0^1 x^2y+\frac{1}{3}y^3 \Big|_{x^2}^{x} dx$$

$$= \int_0^1 \frac{4}{3}x^3 - x^4 - \frac{1}{3}x^6 \, dx$$

$$= \frac{1}{3}x^4 - \frac{1}{5}x^5 - \frac{1}{21}x^7 \Big|_0^1$$

$$= \frac{2}{35}$$