九州大学大学院システム情報科学府 情報理工学専攻 電気電子工学専攻

修士課程　入学試験問題

（令和４年８月２９日）

数 学 (Mathematics)

(6 枚中の 6)

分野毎に解答用紙を別にすること．
Use a separate answer sheet for each field.

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4. 【確率・統計 (Probability and statistics) 分野】

箱の中に５枚のコイン（コイン１～コイン５）がある．箱から一様ランダムにコインを１枚選んで何度か投げる試行を考える．ただし，それぞれのコイン i の表が出る確率 p_i は次の通りである．

$$p_1 = 0,\ p_2 = 1/4,\ p_3 = 1/2,\ p_4 = 3/4,\ p_5 = 1$$

表が出る事象を H とし，コイン i が選ばれる事象を C_i とする．

(1) 選んだコインを１回投げるとする．表が出る確率 $p(H)$ を答えよ．

(2) 選んだコインを１回投げたところ表が出たとする．条件付き確率$p(C_i \mid H)$を $i = 1, \ldots, 5$ についてそれぞれ求めよ．

(3) 選んだコインを２回投げるとする．条件付き確率$p(H_2 \mid H_1)$を求めよ．ただし H_j は j 回目に表が出る事象であり，$j = 1, 2$ である．

(4) 選んだコインを４回投げるとする．$p(C_i \mid B_4)$を $i = 1, \ldots, 5$ についてそれぞれ求めよ．ただし B_4 は４回目に初めて表が出る事象を表す．

A box contains 5 coins (coin $1, \ldots,$ coin 5). Consider a trial in which we select a coin uniformly at random, and toss it for a certain number of times. Let p_i denote the probability of getting a head on each coin i, and they are given as follows:

$$p_1 = 0,\ p_2 = 1/4,\ p_3 = 1/2,\ p_4 = 3/4,\ p_5 = 1$$

Let H denote the event that a head shows up, and let C_i denote the event that coin i is selected.

(1) Select a coin and toss it once. Find the probability of getting a head $p(H)$.

(2) Suppose a head was obtained after tossing the selected coin once. Find the conditional probability $p(C_i \mid H)$ for each $i = 1, \ldots, 5$.

(3) Suppose we toss the selected coin twice. Find the conditional probability $p(H_2 \mid H_1)$. Here H_j $(j = 1, 2)$ means that a head is obtained on the j-th toss.

(4) Suppose we toss the selected coin four times. Find $p(C_i \mid B_4)$ for each $i = 1, \ldots, 5$. Here B_4 means that the first head is obtained on the fourth toss.

(1) $P(H) = \dfrac{p_1 + p_2 + p_3 + p_4 + p_5}{5} = \dfrac{1}{2}$

(2) $P(C_i|H) = \dfrac{P(C_i H)}{P(H)} = \dfrac{P(H|C_i) \cdot P(C_i)}{P(H)}$

$$= \frac{\frac{1}{5} p_i}{P(H)}$$

thus $P(C_1|H) = 0$

$P(C_2|H) = \dfrac{1}{10}$

$P(C_3|H) = \dfrac{1}{5}$

$P(C_4|H) = \dfrac{3}{10}$

$P(C_5|H) = \dfrac{2}{5}$

(3) $P(H_2|H_1) = \dfrac{P(H_1, H_2)}{P(H_1)} = \dfrac{\sum_{i=1}^{5} (H_1, H_2, C_i)}{P(H_1)}$

$$= \frac{\frac{1}{5}(0 \times 0 + \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2} + \frac{3}{4} \times \frac{3}{4} + 1 \times 1)}{\frac{1}{2}}$$

$$= \frac{3}{4}$$

thus $P(H_2|H_1) = \dfrac{3}{4}$

(4) $P(B_4) = \sum_{i=1}^{5} P(B_4, C_i) = \frac{1}{5}\sum_{i=1}^{5} (1-P_i)^3 P_i$

$$= \frac{1}{5}\left[1^3 \times 0 + \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{1}{2}\right)^3 \times \frac{1}{2} + \left(\frac{1}{4}\right)^3 \times \frac{3}{4} + 0^3 \times 1\right]$$

$$= \frac{1}{5}\left[\frac{27}{4^4} + \frac{1}{4^2} + \frac{3}{4^4}\right]$$

$$= \frac{46}{1280}$$

$$= \frac{23}{640}$$

$$P(C_i | B_4) = \frac{P(C_i, B_4)}{P(B_4)} = \frac{\frac{1}{5}(1-P_i)^3 P_i}{P(B_4)}$$

$\Rightarrow P(C_1 | B_4) = 0$

$P(C_2 | B_4) = \frac{27}{46}$

$P(C_3 | B_4) = \frac{8}{23}$

$P(C_4 | B_4) = \frac{3}{46}$

$P(C_5 | B_4) = 0$