

数学 (Mathematics)

(7枚中の3)

6分野のうちから3分野を選び解答すること。選んだ分野毎に解答用紙を別にすること。
Select 3 fields out of the 6 fields and answer the questions. Use a separate answer sheet for each field.

2. 【微分方程式 (Differential equation) 分野】

次の微分方程式の一般解を求めよ。

$$(1) \frac{dy}{dx} + \frac{y}{x} = \frac{1}{1+x^2}$$

$$(2) (\sqrt{xy} - x) \frac{dy}{dx} = -y$$

Find general solutions to the following differential equations.

$$(1) \frac{dy}{dx} + \frac{y}{x} = \frac{1}{1+x^2}$$

$$(2) (\sqrt{xy} - x) \frac{dy}{dx} = -y$$

$$2. (1) \frac{dy}{dx} + \frac{1}{x} \cdot y = \frac{1}{1+x^2}$$

$$y = e^{-\int \frac{1}{x} dx} \left(\int \frac{1}{1+x^2} e^{\int \frac{1}{x} dx} dx + c \right)$$

$$= e^{-\ln|x|} \left(\int \frac{1}{1+x^2} e^{\ln|x|} dx + c \right)$$

$$= \frac{1}{|x|} \left(\int \frac{1}{1+x^2} \cdot |x| \cdot dx + c \right)$$

$$= \frac{1}{x} \left(\int \frac{x}{1+x^2} dx + c \right)$$

$$= \frac{1}{x} \left(\frac{1}{2} \ln(1+x^2) + c \right)$$

$$= \frac{\ln(1+x^2)}{2x} + \frac{c}{x}$$

$$(2) \frac{dy}{dx} = \frac{-y}{\sqrt{xy}-x} = \frac{-\frac{y}{x}}{\sqrt{\frac{y}{x}}-1}, \text{ let } \frac{y}{x} = u, y = ux$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(ux)}{dx} = x \frac{du}{dx} + u$$

$$\Rightarrow x \frac{du}{dx} + u = \frac{-u}{\sqrt{u}-1}$$

$$\Rightarrow \int \frac{\sqrt{u}-1}{-u\sqrt{u}} du = \int \frac{1}{x} dx$$

$$\Rightarrow -\ln u - 2u^{-\frac{1}{2}} = \ln x + c$$

$$\Rightarrow -\ln\left(\frac{y}{x}\right) - \frac{2}{\sqrt{\frac{y}{x}}} = \ln x + c$$